Physics 7B Midterm 1

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(17) Temperature, Thermal Expansion, and Ideal Gas Law

(17.1)

atomic mass(molecular mass) [u]: relative masses of atoms/molecules, numerically same as molar mass

Conversions

$1u = 1.6605 \times 10^{-27}$ kg

$C = \frac{5}{9}(F-32)$

$F = \frac{9}{5}(C) + 32$

$K = C + 273.15$

(17.2)

Temperature [C°/F°/K]: how hot or cold something is

Freezing	Boiling
0° C	100°C
32° F	212° F

(17.3)

Zeroth Law of Thermodynamics: If two systems are in thermal equilibrium w/ a third system, they are in thermal equlibrium w/ each oth

(17.4)

Linear Expansion: $\boxed{\Delta l = \alpha l_0 \Delta T}$ $\boxed{l = l_0(1 + \alpha \Delta T)}$

α ← coefficient of linear expansion $[\frac{1}{C°}]$

ΔT ← $T - T_0$

NOTE: ring expands linearly by diameter

β ← coefficient of volume expansion $[\frac{1}{C°}]$

Volume Expansion: $\boxed{\Delta V = \beta V_0 \Delta T}$

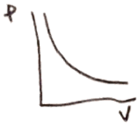
for isotropic: $\Delta V \approx (3\alpha) V_0 \Delta T$

$\boxed{V = l_0(1+\alpha\Delta T) W_0(1+\alpha\Delta T) H_0(1+\alpha\Delta T)}$

(17.6)

Boyle's Law

$V \propto \frac{1}{P}$



Charles's Law

$V \propto T$

Gay-Lussac's Law

$P \propto T$

(17.7)

mole [n]: amount of substance containg same atoms in 12g of C

$$n \text{ (moles)} = \frac{\text{mass (g)}}{\text{molecular mass } (\frac{g}{mol})}$$

Ideal Gas Law: $\boxed{PV = nRT}$ ← temperature [K]

n ← moles

R ← universal gas constant: $8.314 \frac{J}{mol \cdot K} = 1.99 \frac{cal}{mol \cdot K}$

Standard Temperature Pressure (STP): T = 273 K P = 1 atm = 101.3 kPa

(17.9)

Avagadro's number: number of molecules in one mole

$N_A = 6.022 \times 10^{23}$

$\boxed{PV = nN_A kT = NkT = \frac{N}{N_A} RT}$

k ← Boltzmann Constant $(\frac{R}{N_A}) = 1.38 \times 10^{-23} \frac{J}{K}$

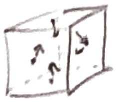
$N = nN_A$

18 Kinetic Theory of Gases

(18.1)

Ideal Gas Law assumptions:

1. Large number of molecules N, mass m, moving in random directions different speeds
2. Molecules on averge far apart from each other
3. Obey classical mechanics, interact when they collide
4. Collisions w/ wall or molecules elastic.



Average translational kinetic energy is proportional to temperature

$$\boxed{\bar{K} = \frac{1}{2} m \bar{v}^2 = \frac{3}{2} kT}$$

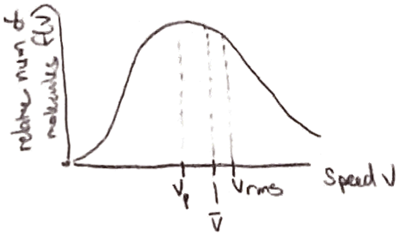
root-mean-square speed: how fast molecules move on avg

$$V_{rms} = \sqrt{\bar{v}^2} = \sqrt{\frac{3kT}{m}}$$

(18.2)

Maxwell Distribution of speeds: probable distribution of speeds in gas w/ N molecules

$$\boxed{f(v) = 4\pi N \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{1}{2}\frac{mv^2}{kT}}} \qquad \boxed{\int_0^\infty f(v)\, dv = N}$$



most probable $V_p = \sqrt{2\frac{kT}{m}} \approx 1.41\sqrt{\frac{kT}{m}}$

avg speed $\bar{V} = \sqrt{\frac{8}{\pi}\frac{kT}{m}} \approx 1.60\sqrt{\frac{kT}{m}}$

rms speed $V_{rms} = \sqrt{3\frac{kT}{m}} \approx 1.73\sqrt{\frac{kT}{m}}$

Expected Value of F

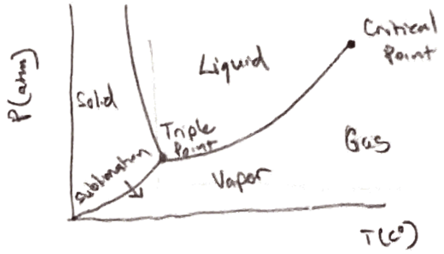
$$\langle F(x) \rangle = \int F(x) p(x)\, dx$$

(18.3)

Liquid crystals between liquid and solid

Sublimation: Solid → vapor

Phase diagram:



(18.4)

Evaporation: liquid → gas

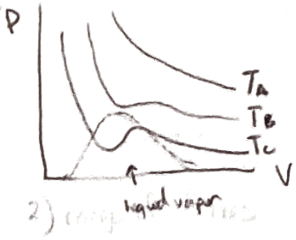
Condensation: gas → liquid

Saturated vapor pressure: equilibrium between liquid, vapor

Boiling when saturated vapor pressure = external pressure

relative humidity: $\frac{\text{partial pressure } H_2O}{\text{saturated vapor pressure } H_2O} \times 100$

Van der Waals



(18.5) Van der Waals takes into account 1) finite size of molecules,
2) range of forces between molecules > size molecules

$$\boxed{P = \frac{RT}{\left(\frac{V}{n}\right) - b} - \frac{a}{\left(\frac{V}{n}\right)^2}} \qquad \boxed{\left(P + \frac{a}{\left(\frac{V}{n}\right)^2}\right)\left(\frac{V}{n} - b\right) = RT}$$

a, b different for different gases

⑲ Heat and First Law of Thermodynamics

(19.1)
Unit of heat: calorie (cal), kilocalorie (kcal): amount of heat necessary to raise 1 g of water by 1°C

heat: energy transferred from one object to another bc difference in temperature

Conversions

1000 cal = 1 kcal = 4184 J
1 L·atm = 101.33 J
F = PA

(19.2)
Internal Energy: sum total of all energy of all molecules in an object

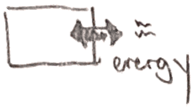
Temperature	vs. Internal Energy	vs. Heat
avg kinetic energy of individual molecules	total energy of all the molecules	transfer of energy from one object to another

Internal Energy (E_{int}): $\boxed{E_{int} = \frac{3}{2} nRT}$ for monotomic gas

(19.3)
Heat Q to change temp of material: $\boxed{Q = mc\Delta T}$

specific heat $\left[\frac{J}{kg \cdot C}\right]$: specific to material

(19.4)
Systems:

Open System	Closed System	Closed (Isolated)
mass can be transferred energy can be transferred	mass cannot be transfered energy can be trasfered	mass cannot be transfered energy cannot be transfered
		

In isolated environments: heat lost = heat gained

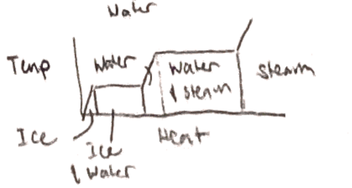
$$Q_1 = Q_2$$

(19.5)
Energy is involved in change of phase

Heat of fusion (L_F): heat to change 1 kg substance solid → liquid
Heat of vaporization (L_V): heat to change 1 kg substance liquid → vapor

$\left[\frac{kJ}{kg}\right]$

$\boxed{Q = mL}$ ← Latent heat



energy is needed to break attractive forces

(19.6)

First Law of Thermodynamics: $\boxed{\Delta E_{int} = Q - W}$

net heat added to system ↗ ↖ net work done by system

Heat added : +
Heat lost : −
Work on System : −
Work by System : +

Thermodynamics:

$$\boxed{\Delta E_{int} = \frac{d}{2} nR\Delta T}$$

degrees of freedom

$$\boxed{W = \int_{V_1}^{V_2} P\,dV}$$

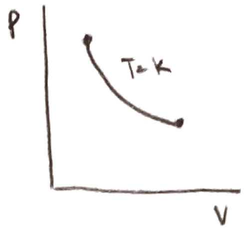
$$\boxed{\Delta E_{int} = Q - W}$$

(19.7)

1) Isothermic $(\Delta T = 0)$

$\Delta E_{int} = 0$, so $Q = W$

$$W = nRT \ln\left(\frac{V_2}{V_1}\right)$$



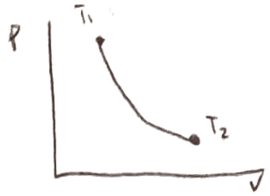
2) Adiabatic $(Q = 0)$

$$\Delta E_{int} = -W = \frac{d}{2} nR\Delta T$$

$$PV^{\gamma} \Rightarrow \frac{P_1}{P_2} = \left[\frac{V_2}{V_1}\right]^{\gamma} = \left[\frac{T_1}{T_2}\right]^{\frac{\gamma}{\gamma-1}}$$

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

Monotomic : $\gamma = 5/3$ $d=3$
Diatomic : $\gamma = 1.4$ $d=5$
Triatomic : $\gamma \approx 1.31$ $d=7$

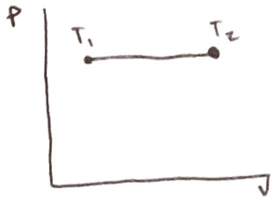


3) Isobaric $(\Delta P = 0)$

if ideal: $W = nRT_2\left(1 - \frac{V_1}{V_2}\right)$

$W = P\Delta V$

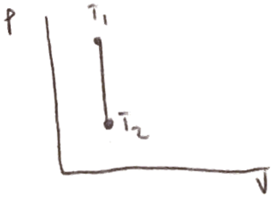
$Q = \Delta E_{int} + P\Delta V$



4) Isovolumetric $(\Delta V = 0)$

$W = 0$

$\Delta E_{int} = Q = \frac{d}{2} nR\Delta T$



Ex) For problems, set up table

	a	b	c
P			
V			
T			

	ab	bc	ca
ΔE_{int}			
Q			
W			

(19.8)

Molar Specific Heats (C_v, C_p): heat required to raise 1mol of gas by 1 C° const volume, temp

Constant Volume

$$\boxed{Q = n\,C_v\,\Delta T}$$

$C_v = M\,c_v$ ← specific heat @ const volume

molecular mass $\left[\frac{g}{mol}\right]$

$\Delta E_{int} = Q$

$C_v = \frac{3}{2} R$

Constant Pressure

$$\boxed{Q = n\,C_p\,\Delta T}$$

$C_p = M\,c_p$

- More heat is required, need work

$Q_p - Q_v = P\Delta V$

$C_p - C_v = R$

(19.10)

- Heat transfer via:

1) Conduction: hot to cold via molecular collisions

$$\boxed{\frac{dQ}{dt} = -kA\frac{dT}{dx}}$$

↑ thermal conductivity constant (specific to metal)

2) Convection: heat flows by mass movement of molecules

3) Radiation: heat by electromagnetic waves

Stephan-Boltzmann eq: $\boxed{\frac{\Delta Q}{\Delta t} = \epsilon \sigma A T^4}$

emissivity: [0,1] characteristic of surface

Stephan-Boltzmann constant·

$\theta = 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$

area of emitting object

Sun radiation heat: $\frac{\Delta Q}{\Delta t} = \left(1000 \frac{W}{m^2}\right) \epsilon A \cos\theta$

(20) Second Law of Thermodynamics

(20.2)

Heat Engines produce work from thermal energy

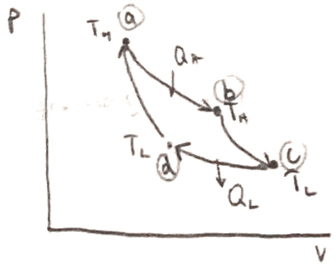
$\boxed{Q_H = W + Q_L}$ $\Delta E_{int} = 0$

Efficiency (e): ratio of work done to heat input

$$\boxed{e = \frac{W}{Q_H} = 1 - \frac{Q_L}{Q_H}}$$

(20.3)

Carnot's Engine is an idealized reversible cycle



ab 1) expanded isothermally, Q_H added

bc 2) expanded adiabatically, temperature reduced to T_L

cd 3) compressed isothermally, Q_L removed

da 4) compressed adiabatically, temperature raised to T_H

$$\boxed{\frac{Q_L}{Q_H} = \frac{T_L}{T_H}} \qquad \boxed{e_{ideal} = 1 - \frac{T_L}{T_H}} \leftarrow [K]$$

20.4

Coefficient of Performance (COP): heat removed for work done
refridgerator

$$COP = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$$

$$COP_{ideal} = \frac{T_L}{T_H - T_L}$$

20.5

Entropy [S] is a state variable, measure of order or disorder

20.6

$$\Delta S = \Delta S_H + \Delta S_L = -\frac{Q}{T_{HM}} + \frac{Q}{T_{LM}}$$ For hot → cold, $\Delta S > 0$

$$\boxed{\Delta S = \frac{Q}{T} = \int_{T_1}^{T_2} \frac{mc\,dT}{T} = mc \ln\left(\frac{T_2}{T_1}\right)}$$

Entropy of isolated system never decreases.

$$\Delta S = \Delta S_{syst} + \Delta S_{env} > 0$$

20.7

Second law of Thermodynamics: Natural Processes tend to move toward a state of greater disorder

20.8

Energy eventually becomes degraded and unavailable to do useful work

Tips
- Remember to change T to Kelvins

Units / Conversions

Force	Newton	[N]	$\frac{m \cdot kg}{s^2}$
Pressure	Pascal	[Pa]	$\frac{kg}{m \cdot s^2}$
Energy	Joule	[J]	$\frac{m^2 \cdot kg}{s^2}$
Energy	Calorie	[cal]	$\frac{m^2 \cdot kg}{s^2}$
Power	Watt	[W]	$\frac{J}{s} = \frac{kg \cdot m^2}{s^3}$
Gas constant		[R]	

$F = PA$ (Pressure) $F = \frac{p}{t}$ (momentum) $p = mv$

$101{,}325\ Pa = 1\ atm = 760\ mmHg = 14.7\ psi$

$1000\ J = 1\ kJ$, $101.33\ J = 1\ L \cdot atm$

$1000\ cal = 1\ kcal$, $1\ cal = 4.184\ J$

$745.7\ W = 1\ hp$, power → $P = \frac{W}{\Delta t}$

$8.314\ \frac{J}{mol \cdot K} = 1.99\ \frac{cal}{mol \cdot K}$

$KE = \frac{1}{2} m v^2$ $\Delta K + \Delta U + \Delta E_{int} = Q - W$

Physics 7B Midterm 2

Jeffrey Shen

(21) Electric Charge and Electric Field

(21.1) Unlike charges attract, like charges repel

Law of conservation of electric charge: net amount of electric charge produced is 0

(21.2) Atom has positively charged nucleus w/ protons and neutrons, electrons surrounding

Becomes an ion if loses or gains an electron

(21.3) Conductors - electrons are bound loosely, charge transfers easily

Insulators - electrons bound tightly to nucleus, charge does not transfer easily

Semiconductor - intermediate category, fewer free electrons

(21.4) Charging by conduction - using charged object to make neutral object charged by contact

Induced charge - caused neutral object to be charged without contact

(21.5) Coulomb's Law:

$$\boxed{F = k\frac{Q_1 Q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}}$$

$k = \frac{1}{4\pi\varepsilon_0}$

Used for Point charges

Magnitude of Electric Force

$k = 8.99 \times 10^9 \; \frac{N \cdot m^2}{C^2}$

permitivity of free space $\varepsilon_0 = 8.85 \times 10^{-12} \; \frac{C^2}{N \cdot m^2}$

Charge (Q) measured in Coulomb [C]

elemtary charge: $e = 1.602 \times 10^{-19}$ C

Principle of superposition - net force on object w/ multiple charges is vector sum of forces due to each of others

(21.6) Each object radiates Electric field, use small positive test charge to measure field

Electric Field

$$\boxed{\vec{E} = \frac{\vec{F}}{q} \qquad E = k\frac{Q}{r^2} \qquad \vec{F} = q\vec{E}}$$

magnitude of test charge

Force at q

Positive charge: E field points away, Negative: points toward

If multiple charges: Superposition Principle: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots$

draw diagram, find mag w/ Coulombs, add vector forces

(21.7) Continous Charge Distribution problems

$\frac{Q}{x} = \lambda \qquad \frac{Q}{A} = \sigma \qquad \frac{Q}{V} = \rho$

1) Choose Coordinate System (Cartesian, Polar, Spherical, Cylindrical)

2) find dq

$dq = \lambda dx$, λ : linear charge density

$dq = \sigma dA$, σ : surface charge density

$dq = \rho dV$, ρ : volume charge density

$dA = dx\,dy = r\,dr\,d\theta$

$dV = dx\,dy\,dz = r^2 \sin\theta \, d\theta \, d\phi \, dr = r\,dr\,dy\,dz$

3) find dE

$$\boxed{dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}}$$

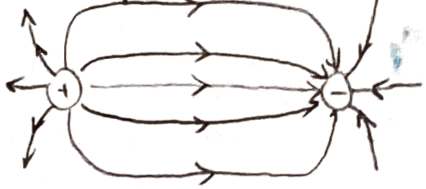
$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2}\hat{r}$

4) find E

$$\boxed{\vec{E} = \int d\vec{E}}$$

(21.8) Electric field lines indicate the direction of electric field at various points.

The closer the lines, the stronger, lines start on pos end on neg



(21.9) Electric Fields and Conductors:

- Electric field inside conductors is 0
- Electric field is always perpendicular to surface outside conductor

(21.10) Magnitude of electron acceleration $a = \frac{F}{m} = \frac{qE}{m}$

(21.11) Electric Dipole - two equal charges w/ opposite signs separated by distance l

Dipole moment: $\vec{p} = Ql$

Torque: $\vec{\tau} = \vec{p} \times \vec{E} = pE\sin\theta$

Work: $W = \int_{\theta_1}^{\theta_2} \tau \, d\theta = pE(\cos\theta_2 - \cos\theta_1)$

22 Gauss's Law

(22.1) Electric flux: electric field passing through area

For uniform electric field $\vec{E}$ through A

Flux → $\boxed{\Phi_E = EA\cos\theta}$

For Not Uniform

$\boxed{\Phi_E = \oint \vec{E} \cdot d\vec{A}}$

flux entering enclosed volume is negative, leaving is positive, nonzero when enclosed charge

(22.2) Gauss's Law: $\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}}$ ← net charge enclosed in surface

Solving w/ Gauss:

1) Find surface S that respects symmetry
2) $\int E \cdot dA = |E| \cdot$ Surface Area of S
3) $Q_{encl} \Rightarrow \int dq = \lambda dx, \sigma dA, \rho dV$

(22.3) If conductor w/ charge Q and inside cavity has charge +q
Must be -q charge on surface of cavity and outer surface with Q+q

23 Electric Potential

(23.1) Electric Potential Energy (U) - conservative force for electrostatic

$$\Delta U = -W = -qEd \qquad [\text{Uniform } \vec{E}]$$

Electric Potential (V) - electric potential energy per unit charge

$$V_a = \frac{U_a}{q} \qquad \Delta V = \frac{U_b - U_a}{q} = \frac{-W_b}{q}$$

Voltage (V) - Potential Difference $1V = 1\ J/C$ measures how much work a given charge can do

$$\Delta U = qV_{ba}$$

(23.2) $\Delta V = -\int \vec{E}\cdot d\vec{l}$ since E is force per unit charge $E = \frac{F}{q}$

(23.3) Electric Potential at distance r away

$$\Delta V = -\int_{r_a}^{r_b} \vec{E}\cdot d\vec{l} = -\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{1}{r^2}\,dr = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r_b}-\frac{Q}{r_a}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$ [single pt charge $V=0$ at $r=\infty$]

(23.4) with contnous distribution

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}}$$ $\boxed{V = -\int E\,dr}$ can add together Voltages since scalar

(23.5)

Equipotential lines with same potential, perpendicular to electric field

(23.6) Electric Dipole Potential

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Ql\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$ [dipole, $r \gg l$]

(23.7)

$$\boxed{E_l = -\frac{dV}{dl}}$$ $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, $E_z = -\frac{\partial V}{\partial z}$

(23.8) Charges moved from $V=0$ $r=\infty$

$$U = Q_2V = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{12}}$$ $$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_2Q_3}{r_{23}}\right)$$

electron Volt (eV) $1\,eV = 1.6\times10^{-19}\,J$

(24) Capacitance, Dielectrics, Electric Energy Storage

(24.1)

Capacitors - Store electric charge by using two conducting objects

capacitor [-||-] battery [-|ı-]

amount of charge acquired by plate $\boxed{Q = CV}$

↑ capacitance [F] $\frac{C}{V}$ Farad

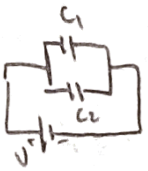
(24.2)

Capacitance $\boxed{C = \varepsilon_0\frac{A}{d}}$ ← area of plates

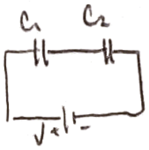
↑ distance between plates

(24.3)

Parallel: $Q = C_{eq}V$ $\boxed{C_{eq} = C_1 + C_2 \cdots}$



Series: $Q = C_{eq}V$ $\boxed{C_{eq} = \frac{C_1C_2}{C_1+C_2}}$



24.4 Harder to charge capacitor the more energy it has

Work to charge $W = \int_0^Q V\,dq = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$

Energy stored $U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$

Energy Density (u) $\frac{\text{energy}}{\text{volume}}$: $u = \frac{1}{2}\varepsilon_0 E^2$ $E = \frac{Q}{\varepsilon_0 A}$

24.5 Dielectric : piece of insulating sheet of material in between plates

$$\boxed{C = K C_0}$$

← capacitance if space is vacuum

Dielectric constant

$C = K\varepsilon_0 \frac{A}{d}$

permitivity of dielectric

$\varepsilon = K\varepsilon_0$

25 Electric Currents and Resistance

25.2 Current only flows with complete circuit

$I = \frac{dQ}{dt}$ Current [I] measured in Amperes [A] $1A = 1\frac{C}{s}$

25.3 Ohm's Law: $\boxed{V = IR}$

Resistance of a wire [Ω] Ohms $1\Omega = 1\frac{V}{A}$

25.4 Resistivity

wire length l

$\boxed{R = \rho\frac{l}{A}}$ $\boxed{dR = \rho\frac{dl}{A}}$ $\sigma = \frac{1}{\rho}$

cross sectional area

resistivity [Ω·m]

conductivity $\left[\frac{1}{\Omega \cdot m}\right]$

resistivity can vary based on temperature

$\rho_T = \rho_0\left[1 + \alpha[T - T_0]\right]$

resistivity Temp T

resistivity at temp T_0

25.5 $\boxed{P = IV = I^2R = \frac{V^2}{R}}$

applies to resistors

Power [W] Watt $1W = 1\,J/s$

Can be measured in kilo watt-hour (kWh) $1\,kWh = 3.6 \times 10^6\,J$

25.7 $V = V_0 \sin(\omega t)$ $\begin{matrix} +V_0 \\ -V \end{matrix}$ peak voltage

$I = I_0 \sin(\omega t)$ $I_0 = V_0/R$ peak current

$P = I^2R = I_0^2 R \sin^2\omega t$

$I_{rms} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}} = 0.707 I_0$ $\qquad V_{rms} = \sqrt{\overline{V^2}} = \frac{V_0}{\sqrt{2}} = 0.707\ V_0$

$$\overline{P} = I_{rms} V_{rms} = \frac{1}{2} I_0^2 R = I_{rms}^2 R = \frac{1}{2}\frac{V_0^2}{R} = \frac{V_{rms}^2}{R}$$

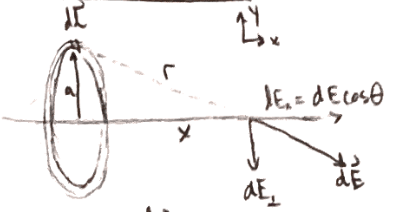
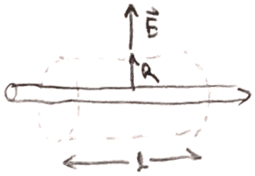
(26.8) $\vec{j}$ current per unit cross-sectional area

$\Delta Q = $ (# charges, N) × (charges per particle)

$= (nV)(-e) = -(nAv_d\,\Delta t)e$

$$I = \frac{\Delta Q}{\Delta t} = -neAv_d$$

$j = \frac{I}{A}$ $\qquad \vec{j} = -ne\vec{v}_d$

	Constant / Example	Reference	
$E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}}$	Coulomb's		$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(x^2+a^2)^{1/2}}$
$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{(z^2+R^2)^{\frac{1}{2}}}\right]$	Coulomb's		$V = \frac{Q}{2\pi\varepsilon_0 R^2}\left[(z^2+R^2)^{1/2} - z\right]$
$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$	Coulombs Gauss		
$r > r_0$: $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ $r < r_0$: $E = \frac{Qr}{4\pi\varepsilon_0 r_0^3}$	Gauss		$r > r_0$: $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$
$E = \frac{\sigma}{2\varepsilon_0}$	Gauss		

Constants

$k = 8.99 \times 10^{9}\ \frac{N \cdot m^2}{C^2}$

$\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{C^2}{N \cdot m^2}$

$e = 1.602 \times 10^{19}\ C$

Surface Area of Sphere: $4\pi r^2$

26 DC Circuits

26.1 Voltage difference when connected → $V_{ab} = \mathcal{E} - Ir$ ← Internal resistance of battery

emf when no current ↗ (ε) ↖ If I current flows from battery

26.2

Series: 

$R = R_1 + R_2$

$V = V_1 + V_2 = IR_1 + IR_2$

Parallel: 

$R = \frac{R_1 R_2}{R_1 + R_2}$ $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2}$

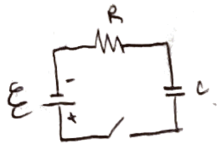
26.3

Kirchoffs first rule (Junction Rule): At Junction Point, sum of currents entering must equal sum of currents leaving Junction

Kirchoffs second rule (Loop Rule): Sum of changes in potential around any closed loop of circuit must be zero

1) Label currents 2) Identify unknowns 3) Apply Junction Rule 4) Apply Loop Rule

26.5



$\mathcal{E} = IR + \frac{Q}{C}$

time to reach 63% charge/Voltage

Charging Capacitor: $Q = C\mathcal{E}(1 - e^{-t/RC})$ $V_c = \mathcal{E}(1 - e^{-t/RC})$ $\tau = RC$

$I = \frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}$

when decreased to 37% charge

Discharging Capacitor: $Q = Q_0 e^{-t/RC}$ $V_c = V_0 e^{-t/RC}$

$I = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}$

27 Magnetism

27.1 Magnets also have magnetic fields surrounding

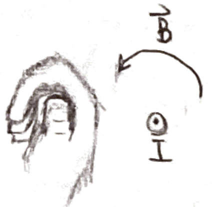
27.2 An electric current produces a magnetic field

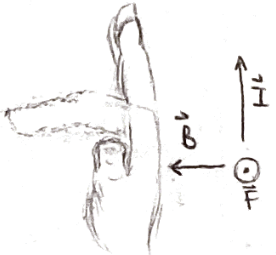
27.3

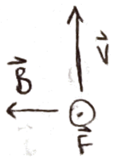
$$\boxed{F = I l B \sin\theta \quad F_{max} = I l B \quad d\vec{F} = I d\vec{l} \times \vec{B}}$$

wire current ↗ ↖ wire length

Magnetic field [B] : unit Tesla [T] $1\,T = 1\,\frac{N}{A \cdot m}$







(27.4)

$$\boxed{\vec{F} = q\vec{v} \times \vec{B} \qquad F = qvB\sin\theta}$$

Force on one of the N particles

velocity of the particle

Centripetal force: $a = \frac{v^2}{r}$

$F = ma$

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

T time for particle charge q moving w/ constant speed v to make one revolution (cyclotron freq)

If there is both magnetic field $\vec{B}$ and electric field $\vec{E}$

Lorentz: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

(27.5) For closed loop of wire in external magnetic field

Torque $\boxed{\tau = NIAB\sin\theta = \vec{\mu} \times \vec{B}}$

N loops of wire

A = a b (area)
a = length vertical arm
b = width of coil

magnetic Dipole moment $\boxed{\vec{\mu} = NI\vec{A}}$

perpendicular to the plane of coil

Potential Energy $\boxed{U = \int \tau \, d\theta = \int NIAB\sin\theta \, d\theta = -\mu B\cos\theta = -\vec{\mu}\,\vec{B}}$

$$\frac{e}{m} = \frac{E}{B^2 r}$$

radius of curvature

Difference in potential due to separation of charge: Hall field $\vec{E}_H$ $\quad E_H = v_d B \quad \mathcal{E}_H = E_H d = v_d B d$

28 Sources of Magnetic Fields

(28.1) Magnetic Field $B = \frac{\mu_0 I}{2\pi r}$ [near long straight wire]

permeability of free space $\mu_0 = 4\pi \times 10^{-7} \, \frac{T \cdot m}{A}$

(28.2) For 2 parallel wires, force on 2nd wire: $F_2 = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} l_2$ [parallel wires]

distance d between wires

length wire 2

(28.3) Coulomb: $1 \, C = 1 \, A \cdot s$

(28.4) Ampère's law: $\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}}$ ← net current passing through surface enclosed by the path

parallel comp of B

- If all points same distance from line, can find B at any point

(28.5) Solenoid: long coil of wire consisting of many loops

Outside solenoid is small enough to be neglible field

Perpendicular segments to $\vec{B}$ are zero

$$B = \mu_0 \frac{N}{l} I = \mu_0 n I \qquad \text{[solenoid]}$$

loops per unit length

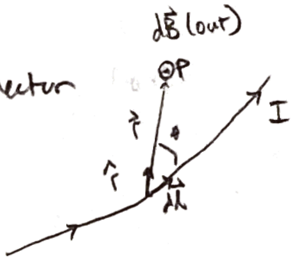
number of loops path encircles

(28.6) Biot-Savart Law: $$\boxed{d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0 I \, dl \sin\theta}{4\pi r^2}}$$

unit vector of the displacement vector from $d\vec{l}$ to point P

infinitesimal length of wire

current I considered as many tiny current elements



$$\boxed{B = \int d\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2}}$$

For magnetic field produced by magnetic dipole along dipole axis

$$B = \frac{\mu_0}{2\pi} \frac{\mu}{(R^2 + x^2)^{3/2}} \qquad \text{[magnetic dipole]}$$

$$B \approx \frac{\mu_0}{2\pi} \frac{\mu}{x^3} \qquad \text{[on axis, magnetic dipole, } x \gg R\text{]}$$

(29) Electromagnetic Induction and Faraday's Law

(29.1) A changing magnetic field can produce electric current or induces an emf

(29.2) Magnetic flux: $\boxed{\Phi_B = \int \vec{B} \cdot d\vec{A}}$ $\qquad \Phi_B = BA\cos\theta = \vec{B} \cdot \vec{A}$ [B uniform]

Faraday's Law of induction: $\mathcal{E} = -\frac{d\Phi_B}{dt}$ $\qquad \mathcal{E} = -N\frac{d\Phi_B}{dt}$ [N loops]

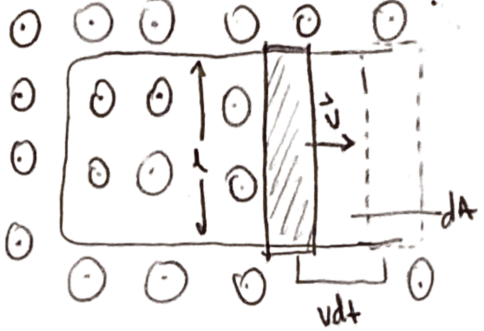
emf induced is equal to rate of change of magnetic flux through circuit

Lenz's Law: induced emf is always in direction opposing original change in flux that caused it

Must have 1) external field whose flux must be changing to induce electrical current
2) magnetic field produced by induced current

magnetic field a) points in same direction as external if flux ↓ b) opposite if ext ↑ c) 0 flux not

(29.5) If U shaped Conductor with moving rod



$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{BdA}{dt} = \frac{Bl v\,dt}{dt} = Blv$$

Force to move rod $\quad F = IlB = \frac{B^2 l^2 v}{R}$

(29.4)

Electric Generators

rotating coil N loops $\quad \mathcal{E} = \underset{\uparrow\,\#\text{ loops}}{N} B \underset{\uparrow\,\text{area of loop}}{A}\,\omega \sin \omega t = \mathcal{E}_0 \sin \omega t \qquad \omega = 2\pi f$

(29.6)

Transformer Eq: $\quad \frac{V_s}{V_p} = \frac{N_s}{N_p}$ ← turns in secondary, ← turns in primary coil $\qquad \frac{I_s}{I_p} = \frac{N_p}{N_s}$

Input → V_p Primary voltage

If $\quad N_s > N_p$: Step up transformer $\quad V_s > V_p$

$\quad N_p > N_s$: Step down transformer $\quad V_p > V_s$

(29.7) A changing magnetic flux produces an electric field

General form Faraday's Law $\quad \boxed{\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}}$

(30) Inductance, Electromagnetic Oscillations, and AC Circuits

(30.1) Two coils of wire near each other with changing current induce an emf in the other

mutual Inductance $\quad M_{21} = \frac{N_2 \Phi_{21}}{I_1}$ ← flux of coil 2 caused by 1

$$\mathcal{E}_2 = -M_{21}\frac{dI_1}{dt} = -M\frac{dI_1}{dt} \qquad \mathcal{E}_1 = -M\frac{dI_2}{dt}$$

(30.2) Cause induction from changing current in own wire

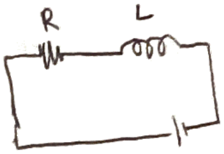
Self Inductance $\quad \boxed{L = \frac{N\Phi_B}{I}} \qquad \mathcal{E} = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$

Inductor with significant Inductance -ooo- [H]

(30.3) Energy stored in inductor $\quad U = \frac{1}{2}LI^2$

energy density $\quad u = \frac{1}{2}\frac{B^2}{\mu_0}$

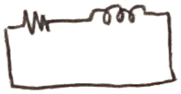
(30.9) LR Circuits



Changing

Current rises and approaches $I_{max} = \frac{V_0}{R_0}$

$L \frac{dI}{dt} + RI = V_0$ $\quad I = \frac{V_0}{R}(1 - e^{-t/\tau})$ $\quad \tau = \frac{L}{R}$ time for current to reach 63%



Discharging

$I = I_0 e^{-t/\tau}$

(30.5) LC Circuits

capacitor discharges

$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0$ $\quad Q = Q_0 \cos(\omega t + \phi)$ $\quad \omega = 2\pi f = \sqrt{\frac{1}{LC}}$

$$I = -\frac{dQ}{dt} = \omega Q_0 \sin(\omega t + \phi)$$
$$= I_0 \sin(\omega t + \phi)$$

Charge in LC capacitor oscillates

$$I_{max} = \omega Q_0 = \frac{Q_0}{\sqrt{LC}}$$

Energy $U = \frac{Q_0^2}{2C}$ $\quad$ LC Oscillator or electromagnetic oscillation

Constants

$\mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$