Math 53 Midterm 1 Study Guide

Chapter 10: Parametric Equations and Polar Coordinates

10.1 Curves Defined by Parametric Equations

Parametric equations: $x = f(t)$ $y = g(t)$

- Know how curves are drawn and direction, how much

10.2 Calculus with Parametric Curves

Tangents: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ if $\frac{dx}{dt} \neq 0$

2nd Derivative (Concavity): $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt}$

Area: $A = \int_\alpha^\beta g(t) f'(t)\,dt$ if $x = f(t)$, $y = g(t)$, $\alpha \leq t \leq \beta$

Arc Length: $L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Surface Area: $S = 2\pi \int_\alpha^\beta y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$ if rotated around x-axis

10.3 Polar Coordinates

Polar coordinate system: (r, θ)

$x = r\cos\theta$	$y = r\sin\theta$	rect → polar
$r^2 = x^2 + y^2$	$\tan\theta = \frac{y}{x}$	polar → rect

Polar graphs

$r = c$: circle $\qquad \theta = c$: line

Cardioid/Limacon: $r = a \pm b\cos\theta$, $r = a \pm b\sin\theta$

Inner loop: $b > a$ $\qquad$ Cardiod (heart): $a = b$

Roses: $r = a\cos(n\theta)$ $\qquad r = a\sin(n\theta)$

if n is odd: petals $= n$ $\qquad$ even: petals $= 2n$

Tangents:
$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$
since $x = r\cos\theta$, $y = r\sin\theta$

10.4 Areas and Lengths in Polar Coordinates

Area: $A = \frac{1}{2}\int_a^b r^2\, d\theta$

Arc Length: $L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

Chapter 12: Vectors and the Geometry of Space

12.1 3D Systems

Distance: $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

Eq of sphere: $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$

12.2 Vectors

vector has both magnitude and direction

$$\vec{a} = \langle x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1 \rangle$$

Magnitude: $|a| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

$a + b = \langle a_1 + b_1, a_2 + b_2 \rangle$ $\qquad a - b = \langle a_1 - b_1, a_2 - b_2 \rangle$ $\qquad ca = \langle ca_1, cb_1 \rangle$

$\vec{a} = |a| \langle \cos\theta, \sin\theta \rangle$

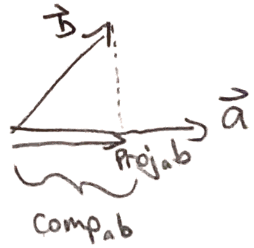
12.3 Dot Product

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}$ where θ is the angle between vectors $\vec{a}$ and $\vec{b}$

if $\vec{a} \cdot \vec{b} = 0$, vectors are orthogonal

Projections



Scalar proj of b onto a

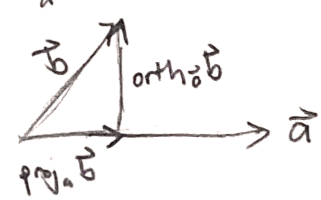
$\text{comp}_a b = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$

Vector proj of b onto a

$\text{proj}_a b = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a}$

Orthogonal Vector

$\text{Orth}_{\vec{a}} \vec{b} = \vec{b} - \text{proj}_{\vec{a}} \vec{b}$



$\text{Work} = F \cdot D \cos\theta = \vec{F} \cdot \vec{D}$

12.4 Cross Product

$\text{Det} \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$$a \times b = \det \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}$$

$\vec{a} \times \vec{b}$ is orthogonal to both $\vec{a}$ and $\vec{b}$

$|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}| \sin\theta$

if $\vec{a} \times \vec{b} = 0$, vectors $\vec{a}$ and $\vec{b}$ are parallel

$|\vec{a} \times \vec{b}|$ is equal to parallelogram made by $\vec{a}$ and $\vec{b}$

Scalar triple product : $\vec{a} \cdot (\vec{b} \times \vec{c})$

(Volume of parallelpiped) = $|\vec{a} \cdot (\vec{b} \times \vec{c})|$

Torque : $\vec{T} = \vec{r} \times \vec{F}$ $\quad |T| = |\vec{r} \times \vec{F}| = |\vec{r}||\vec{F}| \sin\theta$

12.5 Equations of Lines and Planes

vector eq of Line: $\vec{r} = \vec{r}_0 + t\vec{v}$ where $\vec{v}$ is a direction vec and $\vec{r}_0$ is a vector on the line

parametric eqs : $x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$

Symmetric eqs : $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

Planes

Scalar eq of plane : $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ where $\langle a, b, c \rangle$ is a vector normal to plane

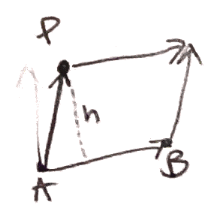
linear eq of plane : $ax + by + cz + d = 0$

- Use normal vectors to find angle between planes

Distance from point to plane : $D = \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$ where (x_1, y_1, z_1) is point and plane $ax + by + cz + d = 0$

$$D = \frac{|\vec{n} \cdot \vec{b}|}{|\vec{n}|}$$

Distance from a point to line : $h = \dfrac{|\vec{AB} \times \vec{AP}|}{|\vec{AB}|}$ $\quad \dfrac{\text{(area of parallelogram)}}{\text{(base)}}$



Intersections

Check is direction vectors are parallel (scalar multiples)

Set $x_1 = x_2$, $y_1 = y_2$, $z_1 = z_2$ in terms of t and s

12.6 Cylinders and Quadric Surfaces

Cylinder is a surface consisting of all lines through plane

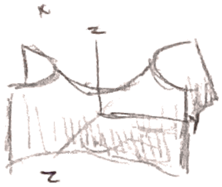
Quadric Surfaces

$$Ax^2+By^2+Cz^2+Dxy+Eyz+Fxz+Gx+Hy+Iz+J=0$$

Use traces, setting $z, y, x = c$ and graph

Ellipsoid : $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ — traces are ellipses; $a=b=c$ sphere

Elliptic Paraboloid : $\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ — H traces are ellipses; V traces are parabolas



Hyperbolic Parabolid : $\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$ — H traces are hyperbolas; V traces are parabolas

Cone : $\frac{z^2}{c^2}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ — H traces are ellipses; V traces $x=k$ hyperbolas; $k\neq 0$ pairs of lines

Hyperboloid of One Sheet : $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ — H traces ellipses; V traces Hyperbolas

Hyperboloid of Two Sheets : $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ — H traces in $z=k$ ellipses; V traces are hyperbolas

Chapter 13: Vector Functions

13.1 Vectors Functions and Space Curves

$$r(t)=\langle f(t), g(t), h(t)\rangle$$

$$\lim_{t\to a}\vec{r}(t)=\left\langle \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t)\right\rangle$$

13.2 Derivatives and Integrals of Vector Func.

$$\vec{r}\,'(t) = \langle f'(t), g'(t), h'(t) \rangle$$

Unit Tangent Vector: $\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$

$$\int_a^b \vec{r}(t)\,dt = \left\langle \int_a^b f(t)\,dt, \int_a^b g(t)\,dt, \int_a^b h(t)\,dt \right\rangle$$

13.3 Arc Length and Curvature

Arc Length: $s = \displaystyle\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt = \int_a^b |\vec{r}\,'(t)|\,dt$

$$\frac{ds}{dt} = |r'(t)|$$

<u>Reparametrize in terms of s:</u>

$r(t) \Rightarrow r(s)$

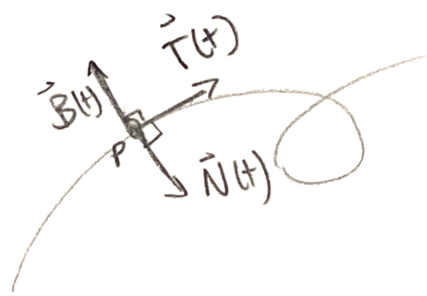
1. Solve for s
2. use eq to solve for t in terms of s
3. Plug t equation into $\vec{r}(t)$

Curvature: how quickly curve changes direction

$r = \frac{1}{K}$ of osculating circle

$$K = \frac{|\vec{T}\,'(t)|}{|r'(t)|} \quad \text{or} \quad \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$$

<u>Normal and Binormal vectors</u>



$$\vec{N}(t) = \frac{\vec{T}\,'(t)}{|\vec{T}\,'(t)|}$$

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$

The vectors $\vec{N}(t)$ and $\vec{T}(t)$ are on the osculating plane

13.4 Motion in Space: Velocity and Acceleration

position: $\vec{r}$

velocity: $\vec{v} = r'(t)$

acceleration: $\vec{a} = r''(t) = v'(t)$

Force: $\vec{F} = m\vec{a}(t)$

Projectile Motion

$a = \langle 0, -9.8 \rangle$ $\quad v(0) = v_0 \quad$ α = angle

$$r(t) = \langle (v_0 \cos\alpha)t, (v_0 \sin\alpha)t - \tfrac{1}{2}gt^2 \rangle$$

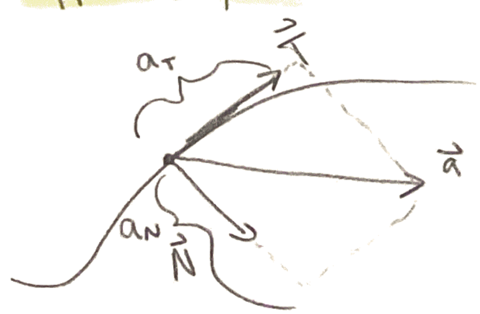
$$d = \frac{v_0^2 \sin 2\alpha}{g} = \frac{2v_0^2 \sin\alpha \cos\alpha}{g}$$

Acceleration

$$\vec{a} = v'\vec{T} + \kappa v^2 \vec{N}$$ where $v = |\vec{v}|$

$a_T = v'$ $\qquad a_N = \kappa v^2$

$$a_T = \frac{\vec{r}\,'(t) \cdot \vec{r}\,''(t)}{|\vec{r}\,'(t)|} \qquad a_N = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|}$$



Math 53 Midterm 2

Chapter 14: Partial Derivatives

14.1 Functions of Several Variables

- function f of two vars $f(x,y)$
- Domain is region in xy-plane, range is z, $f(x,y)$
- Level curves are curves with equations $f(x,y)=k$ where k is constant

14.2 Limits and Continuity

$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$

- If $f(x,y)\to L_1$ as $(x,y)\to(a,b)$ along path C_1 and $f(x,y)\to L_2$ as $(x,y)\to(a,b)$ on path C_2 where $L_1 \neq L_2$ lim DNE

Approach from different lines

Ex) $x=0$ $\lim_{(0,y)} f(x,y) = L$ | $y=x$ $\lim_{(x,x)} f(x,y) = L$ | $y=mx$ $\lim_{(x,mx)} f(x,y) = L$

- function is continuous at (a,b) if $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$

continuous on D if f is continuous at every point (a,b) in D

14.3 Partial Derivatives

$$f_x(x,y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f(x,y) \qquad f_y(x,y) = f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y} f(x,y)$$

Clairaut's Theorem: $f_{xy}(a,b) = f_{yx}(a,b)$

Laplace Equation: $u_{xx} + u_{yy} = 0$

14.4 Tangent Planes and Linear Approximations

Eq of tangent plane: of surface $z=f(x,y)$ at point $P(x_0,y_0,z_0)$

$$z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

- If partials exist near (a,b) and are continuous at (a,b) then f is differentiable at (a,b)

$$L(x,y) = f_x(x-x_0) + f_y(y-y_0) + z_0$$

- $dw = W_x\,dx + W_y\,dy + W_z\,dz$

$\boxed{14.5}$ Chain Rule

① $z = f(x,y)$ where $x = g(t)$ $y = h(t)$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

Case 2: $z = f(x,y)$ $x = g(s,t)$ $y = h(s,t)$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} \qquad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad \frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} \qquad \frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}$$

$\boxed{14.6}$ Directional Derivatives and the Gradient Vector

If f is differential function of x and y f has directional derivative in direction of unit vector $\vec{u} = \langle a, b\rangle$

$$D_u f(x,y) = f_x(x,y)\,a + f_y(x,y)\,b = \nabla f \cdot \vec{u}$$

Gradient: fastest increase

$$\nabla f(x,y) = \langle f_x(x,y), f_y(x,y)\rangle = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right\rangle$$

- Max of Directional Derivative $D_u f(\vec{x})$ is $|\nabla f(\vec{x})|$ when $\vec{u}$ has same direction as gradient $\nabla f(x)$

Tangent Plane

Gradient is $\vec{n}$ of tangent plane

Plane: $F_x(x-x_0) + F_y(y-y_0) + F_z(z-z_0) = 0$

$\boxed{14.7}$ Max and Min Values

- If f has a local max or min at (a,b) the first order partials then $f_x(a,b) = 0$ and $f_y(a,b) = 0$
- A point (a,b) is a critical point if $f_x(a,b) = 0$ and $f_y(a,b)$ or one of partials are DNE

If (a,b) is a critical point

$D = f_{xx}(a,b) f_{xy}(a,b) - [f_{xy}(a,b)]^2$

1. $D>0$ and $f_{xx}(a,b) > 0$, $f(a,b)$ is local min
2. $D>0$ and $f_{xx}(a,b) < 0$, $f(a,b)$ is local max
3. $D<0$, $f(a,b)$ not local min or max (saddle point)
4. $D=0$, no information, could be any

If f is continuous on closed, bounded set D in $\mathbb{R}^2$ there is max $f(x_1,y_1)$ and min $f(x_2,y_2)$ in D

Steps to find max, min

1. Find vals of f at critical points of f in D
2. Find extreme values of f on boundary of D
3. largest is max, smallest is min

$\boxed{14.8}$ Lagrange Multipliers

To find min max of $f(x,y,z)$ constraint $g(x,y,z)=k$

$\nabla f(x,y,z) = \lambda \nabla g(x,y,z)$ $g(x,y,z)=k$

$f_x = \lambda g_x$ $f_y = \lambda g_y$ $f_z = \lambda g_z$ $g(x,y,z)=k$

Two Constraints

$$\nabla f(x_0,y_0,z_0) = \lambda \nabla g(x_0,y_0,z_0) + \mu \nabla h(x_0,y_0,z_0)$$

Chapter 15: Multiple Integrals

$\boxed{15.1}$ Double Integrals over Rectangles

$V = \iint_R f(x,y)\, dA$

Avg value $= \dfrac{\iint_R f(x,y)\, dA}{\text{area of } R} = \dfrac{\text{Volume total}}{\text{Area of base}}$

15.2 Double Integrals over General Regions

- Make sure each (partial or single) integral is doable
- If $m \le f(x,y) \le M$ for all (x,y) in D, then

$$m\,A(D) \le \iint_D f(x,y)\,dA \le M A(D)$$

15.3 Double Integrals in Polar

$r^2 = x^2 + y^2$ $x = r\cos\theta$ $y = r\sin\theta$

$$\iint_R f(x,y)\,dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\; r\,dr\,d\theta$$

15.4 Applications of Double Integrals

Mass : $m = \iint_D \rho(x,y)\,dA$ $\rho(x,y)$ is lamina func

Moment: $M_x = \iint_D y\,\rho(x,y)\,dA$ $M_y = \iint_D x\,\rho(x,y)\,dA$

Center of mass : $\bar{x} = \frac{M_y}{m} = \frac{1}{m}\iint x\,\rho(x,y)\,dA$ $\bar{y} = \frac{M_x}{m} = \frac{1}{m}\iint y\,\rho(x,y)\,dA$

Moment of Inertia

$I_x = \iint_D y^2 \rho(x,y)\,dA$ $I_y = \iint_D x^2 \rho(x,y)\,dA$

I_0 (around origin) $= \iint_D (x^2+y^2)\,\rho(x,y)\,dA = \iint_D r^2 \rho(x,y)\,dA$

Probability

- Probability isn't negative scale 0-1

$f(x,y) \ge 0$ $\iint_R f(x,y)\,dA = 1$

15.6 Triple Integrals

$\iiint_E f(x,y,z)\,dV$ $V(E) = \iiint_E dV$

15.7 Cylindrical Coordinates Triple Integrals

$x = r\cos\theta \quad y = r\sin\theta \quad z = z \quad | \quad r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x} \quad z = z$

$$\iiint_E f(x,y,z)\,dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r\cos\theta,\, r\sin\theta)}^{u_2(r\cos\theta,\, r\sin\theta)} f(r\cos\theta, r\sin\theta)\; r\,dz\,dr\,d\theta$$

15.8 Spherical Coordinates Triple Integrals

$\rho \geq 0 \quad , \quad 0 \leq \phi \leq \pi$

$x = \rho\sin\phi\cos\theta \quad y = \rho\sin\phi\sin\theta \quad z = \rho\cos\phi \quad \rho^2 = x^2 + y^2 + z^2$

$$\iiint_E f(x,y,z)\,dV = \int_c^d \int_\alpha^\beta \int_a^b f \; \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$$

Math 53 Final

15.9 Change of Variables in Multiple Integrals

$$\iint_R f(x,y)\, dA = \iint_S f(x(u,v), y(u,v)) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\, dv$$

Jacobian: $x = g(u,v)$ $y = h(u,v)$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

1. Find $u =$, $v =$ suitable
2. Find $x =$, $y =$
3. Find Jacobian and plug in abs value
4. Plug in to equation

Chapter 16: Vector Calculus

16.1 Vector Fields

Vector Field assigns to each point (x,y) in D vector $F(x,y)$

Gradient Field $\nabla f(x,y) = \langle f_x(x,y), f_y(x,y) \rangle$

Conservative if some f that $F = \nabla f$

f is potential function of F

16.2 Line Integrals

Line Integral of F along C arc length

$$\int_C F \cdot dr = \int_a^b F(r(t)) \cdot r'(t)\, dt = \int_C F \cdot T\, ds$$

$$\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

16.3 Fundamental Theorem for Line Integrals

Thm: Let C be smooth curve $\vec{r}(t)$ $a \le t \le b$
f be differentiable func whose ∇f is continous on C

$$\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$

Independent of path if $\int_{C_1} F \cdot dr = \int_{C_2} F \cdot dr$ for $\int_C F \cdot dr$
conservative vector field depends only on intial and terminal point

Thm: F is vector field continous on open (doesn't contain boundary points) connected (any 2 points can be joined by path) region D. If $\int_C F \cdot dr$ is independent of path,
then is f that $\nabla f = F$

Thm:

Condition
1. Open simply-connected (no holes) region D
2. P and Q have continous first partials
3. $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ throughout D

Then
$\vec{F}$ is conservative

Thm: $\int_C F \cdot dr$ is independent of path in D if and only if $\int_C F \cdot dr = 0$ for every closed path C in D

16.4 Green's Theorem

Positive orientation is counter clockwise

Thm:

Conditions

1. C is positively oriented
2. piecewise-smooth
3. simple closed curve
4. P and Q have continous partial derivatives on open region

Then!

let D be region bound by curve C

$$\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

16.5 Curl and Divergence

$$\text{curl}\,\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \nabla \times F$$

Thm: If fn of 3 vars has continous second-partials,

curl $(\nabla f) = 0$

Thms:

$\vec{F}$ conservative	$\Rightarrow$	curl $(F) = 0$
curl$(F) \neq 0$	$\Rightarrow$	$\vec{F}$ not conservative
curl $(F) = 0$ & simply connected & continous partial derivatives	$\Rightarrow$	$\vec{F}$ conservative
$F = \langle P, Q, R \rangle$ have second partials	$\Rightarrow$	div(curl(F)) = 0
div $(F) \neq 0$	$\Rightarrow$	$\vec{F}$ is not a curl

div $F = \nabla \cdot F$

16.6 Parametric Surfaces and their Areas

Parametric Surface : $r(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$

Surface of Revolution (about x-axis) : $x = x$ $\quad y = f(x)\cos\theta \quad z = f(x)\sin\theta$

Tangent plane contains r_u and r_v vectors and

Normal vector to tangent plane is $r_u \times r_v$

Surface Area :

$$A(S) = \iint_D \| r_u \times r_v \| \, dA$$

or

$$A(S) = \iint_D \sqrt{1 + \left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2} \, dA$$

16.7 Surface Integrals

Surface Integral of f over surface S

$$\iint_S f(x,y,z)\,dS = \iint_D f(r(u,v))\,|r_u \times r_v|\,dA$$

$$\iint_S f(x,y,z)\,dS = \iint_D f(x,y,g(x,y))\sqrt{\left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2 + 1}\,dA$$

Surface Integral of F over S (Flux of F across S)

$$\iint_S F \cdot dS = \iint_S F \cdot n \, dS$$

$$\iint_S F \cdot dS = \iint_D F \cdot (r_u \times r_v)\,dA$$

$$\iint_S F \cdot dS = \iint_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right) dA$$

16.8 Stokes' Theorem

Thm: Conditions:

1. S is piecewise smooth surface
2. Bounded by simple, closed piecewise-smooth boundary curve C w/ positive orientation
3. Continous partial derivatives

Then:

$$\int_C F \cdot dr = \iint_S \text{curl}\, F \cdot dS$$

16.9 Divergence Theorem

Thm: Conditions:

1. E is a simple solid region
2. S be the boundary surface of E positive outward orientation
3. continous partial derivaties on an open region contains E

Then:

$$\iint_S F \cdot dS = \iiint_E \text{div}\, F \, dV$$